NDE 4704-01

Utilizing the Sequential Probability Ratio Test for Building Joint Monitoring

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ABSTRACT

In this application of the statistical pattern recognition paradigm, a prediction model of a chosen feature is developed from the time domain response of a baseline structure. After the model is developed, subsequent feature sets are tested against the model to determine if a change in the feature has occurred. In the proposed statistical inference for damage identification there are two basic hypotheses; (1) the model can predict the feature, in which case the structure is undamaged or (2) the model can not accurately predict the feature, suggesting that the structure is damaged. The Sequential Probability Ratio Test (SPRT) develops a statistical method that quickly arrives at a decision between these two hypotheses and is applicable to continuous monitoring. In the original formulation of the SPRT algorithm, the feature is assumed to be Gaussian and thresholds are set accordingly. It is likely, however, that the feature used for damage identification is sensitive to the tails of the distribution and that the tails may not necessarily be governed by Gaussian characteristics. By modeling the tails using the technique of Extreme Value Statistics, the hypothesis decision thresholds for the SPRT algorithm may be set avoiding the normality assumption. The SPRT algorithm is utilized to decide if the test structure is undamaged or damaged and which joint is exhibiting the change.

Keywords: Damage detection, time series analysis, sequential probability ratio test, extreme value statistics, statistical pattern recognition, vibration test

1. INTRODUCTION

Structural Health Monitoring of building joints is motivated by both life safety and economic concerns. A monitoring algorithm that can accurately disseminate deterioration in a building joint has the potential for saving time and money in the inspection process of large buildings that have undergone stressful conditions. Such an algorithm implemented in a constant monitoring fashion may also provide insight into the health of a building as it ages, alerting owners to potentially dangerous changes in the joints of the structure and allowing maintenance before life threatening critical failure.

Many current approaches to this problem involve methods that leave much to the interpretation of the system operator. These methods may enable a trained eye to discern and locate damage but are not easily automated or quantified. In an attempt to find approaches that can be automated and quantified, the Sequential Probability Ratio Test (SPRT) is proposed to make a statistical decision on the health of a building joint. Such a test returns one of three results: the feature is similar to the original condition, the feature has deviated significantly from the original condition, or that there is not yet enough information to make a decision. Using time history analysis of vibration data, a SPRT is run on a set of chosen features. In this case the features are extracted from the data sets by applying an Auto Regressive - Auto Regressive model with Exogenous inputs (AR-ARX) and computing residual error terms.

Statistical tests, such as the SPRT, are often based on assumptions of normality for the parent distribution. This normality assumption, however, may place misleading constraints on the extreme values of the distribution. These extreme values are where most of the interesting features for damage detection are located, therefore it is important to model them correctly. By re-formulating the SPRT to accurately model the extreme values of the data, accuracy and quickness of the decisions may be increased.

Described within this report are the development of the sequential probability ratio test and the merger of sequential testing with the field of extreme value statistics. Test data came from a three-story test structure with bolted connections between floors and columns to simulate a building joint. The data are then analyzed using both a SPRT formulated with normality assumptions and extreme value distributions. The two tests are then compared.

2. METHODOLOGY

2.1. Sequential test

A sequential statistical inference starts with observing a sequence of random variables $\{x_i\}$ $(i = 1, 2, \ldots)$. This accumulated data set at stage n is denoted as:

$$X_n = (x_1, \ldots\ x_n) \tag{1}$$

The goal of a statistical inference is to reveal the probability model of X_n, which is assumed to be at least partially unknown. When the statistical inference is cast as a parametric problem, the functional form of X_n is assumed known and the statistical inference poses some questions regarding the parameters of the probability model. For instance, if $\{x_i\}$ are independent and identically distributed (i.i.d.) normal variables, one may pose some statistical test about the mean and/or the variance of this normal distribution.

A sequential test is one of the simplest tests for such a statistical inference where the number of samples required before reaching a decision is not determined in advance. An advantage of the sequential test is that on average a smaller number of observations are needed to make a decision compared to the conventional fixed-sample size test. First, a simple hypothsis test containing only two distinct distributions is considered. Here, the interest is in discriminating two simple hypotheses:

$$H_o : \theta = \theta_o, \quad H_1 : \theta = \theta_1, \quad \theta_o \neq \theta_1 \tag{2}$$

where θ is a particular parameter value in question, and it is assumed that θ can take either θ_o or θ_1 only. A *type I error* arises if H_o is rejected when in fact it is true. *Type II errors* arise if H_o is accepted when it is false. When a sequence of observations $\{x_i\}$ are available, the purposes of any sequential test for the above hypotheses are (1) to reach the correct decision about H_o with the least probability of type I and II errors, (2) to minimize the sampling number before the correct decision is made, and (3) to eventually terminate with either the acceptance or rejection of H_o as the sampling size n increases. When a sequential test satisfies the last condition, the test is defined *closed*. Otherwise, an *open* test may continue infinitely observing data without reaching any terminal decision about H_o.

It turns out that the simultaneous achievment of all three goals is impossible by any test. Therefore, a reasonable compromise among the these conflicting goals needs to be achieved. For the well-established fixed-sampling tests, the sample size n is fixed, and an upper bound on the type I error is pre-specified. Then, an optimal fixed-sample test is selected by minimizing the probability of type II error. On the other hand, a sequential test specifies upper bounds on the probabilities of type I and II errors and minimizes the following average sample number, $\mathrm{E}(n \mid \theta)$:

$$E(n \mid \theta) = \sum_{n=1}^{\infty} n\ p(n \mid \theta) \tag{3}$$

where $p(n \mid \theta)$ is the probability mass function of n when θ is the true value of the parameter. Note that for a closed test $p(n < \infty \mid \theta) = 1$ for $\theta = \theta_o$ or θ_1.

There exist a class of sequential tests, and sequential tests, which satisfy the following criteria are called *valid* (Ghosh, 1970):

$$\begin{aligned}&\text{The test is closed.}\\ &1 - Q(\theta) \le \alpha \ \text{ for } \theta = \theta_o \\ &Q(\theta) \le \beta \ \text{ for } \theta = \theta_1\end{aligned} \tag{4}$$

where α and β are the preassined type I and II errors, respectively. $Q(\theta)$ is the probability that any sequential test accepts H_o as $n \to \infty$. In other words,

$$Q(\theta) = \sum_{n=1}^{\infty} \int_{X_n \in R_n^o} f(X_n \mid \theta)\, dX_n \tag{5}$$

The integral in Equation (5) is evaluated over the acceptance region of H_o ($X_n \in R_n^o$). The second criterion in Equation (4) states that for all values of n, the true type I error, $1 - Q(\theta_o)$, should be less than the pre-assigned risk α. In a similar fashion, the third criterion indicates that the true type II error $Q(\theta_1)$ should be less than β. Among various valid sequential tests, it can be proven that the SPRT minimizes on average the sample size required to make a correction making it an optimal sequential test. Because of this extreme sensitivity of the SPRT to signal disturbance, the SPRT has been applied for the surveillance of nuclear power plant components (Humenik and Gross, 1991).

When implementing the SPRT, a trade-off must be considered before assigning values for α and β. When there is a large penalty associated with false positive alarms (for example, alarms that shut down traffic over a bridge), it is desirable to keep α smaller than β. On the other hand, for safety critical systems such as nuclear power plants, one might be more willing to tolerate a false positive alarm to have a higher degree of safety assurance. In this case, β is often specified larger than α.

2.2. Sequential probability ratio test

A SPRT, S(*b*,*a*), for the hypothesis test in Equation (2) is defined as follows (Ghosh, 1970):

Observe a sequence of observations $\{x_i\}$ $(i = 1, 2, \ldots)$ successively, and at stage n;

$$\begin{aligned}&\text{Accept } H_o \text{ if } Z_n \le b\\ &\text{Reject } H_o \text{ if } Z_n \ge a\\ &\text{Continue observing data if } b \le Z_n \le a\end{aligned} \tag{6}$$

where the transformed random variable Z_n is the natural logarithm of the probability ratio at stage n:

$$Z_n = \ln \frac{f(X_n \mid \theta_1)}{f(X_n \mid \theta_o)} \ \text{ for } n \ge 1 \tag{7}$$

thus, testing a probability ratio for sequential observations.

Without any loss of generality, Z_n is defined zero when $f(X_n | \theta_1) = f(X_n | \theta_o) = 0$. b and a are the two stopping bounds for accepting and rejecting H_o, respectively, and they can be estimated by the following Wald approximations (Wald, 1947):

$$b \cong \ln \frac{\beta}{1-\alpha} \text{ and } a \cong \ln \frac{1-\beta}{\alpha} \tag{8}$$

Although closed form solutions of a and b are available for several probability models, it has been a standard practice to employ Equation (8) to approximate the stopping bounds in all practical applications. The continuation region $b \leq Z_n \leq a$ is called the *critical inequality* of S(b,a) at stage n.

In many practical problems, it is often more realistic to formulate the hypothesis test as discrimination between two one-sided hypotheses:

$$H_o : \theta \leq \theta_o, \quad H_1 : \theta \geq \theta_1, \quad \theta_o < \theta_1 \tag{9}$$

The criteria in Equation (4) are now equivalent to

$$\begin{aligned} &\text{The test is closed.} \\ &1 - Q(\theta) \leq \alpha \text{ for } \theta \leq \theta_o \\ &Q(\theta) \leq \beta \text{ for } \theta \geq \theta_1 \end{aligned} \tag{10}$$

Ghosh (1970) shows that the previous SPRT shown in Equation (6) also provides an optimal solution to this hypothesis test defined in Equation (9).

2.3. Applications to Normal Distribution

In the damage detection problem presented, the main interest is to examine how the probability distribution function of the residual errors broadens as data are recorded under a damaged condition of a system. Therefore, the following hypothesis test is constructed using the standard deviation of the residual errors as the parameter in question:

$$H_o : \sigma \leq \sigma_o, \quad H_1 : \sigma \geq \sigma_1, \quad 0 < \sigma_o < \sigma_1 < \infty \tag{11}$$

Here, when the standard deviation of the residual errors, σ, is less than a user specified standard deviation value σ_0, the system in question is considered undamaged. On the other hand, when σ becomes equal to or larger than the other user specified standard deviation σ_1, the system is suspected to be damaged. It should be noted that the selection of σ_0 and σ_1 is structure dependent, and it might be necessary to use signals from a few damage cases in order to establish these two decision boundaries.

If modified observations $\{z_i\}$ $(i = 1, 2, \ldots)$ are defined as follows;

$$z_1 = \ln \frac{f(X_1 | \sigma_1)}{f(X_1 | \sigma_o)} \text{ and } z_i = \ln \frac{f(X_i | \sigma_1) f(X_{i-1} | \sigma_o)}{f(X_i | \sigma_o) f(X_{i-1} | \sigma_1)} \tag{12}$$

then, Z_n becomes:

$$Z_n = \sum_{i=1}^{n} z_i \tag{13}$$

Assuming that X_n has a normal distribution with mean μ and standard deviation σ, z_i can be related to x_i:

$$z_i = \frac{1}{2}(\sigma_o^{-2} - \sigma_1^{-2})(x_i - \mu)^2 - \ln\frac{\sigma_1}{\sigma_o} \tag{14}$$

In a graphical representation of a SPRT S(*b*,*a*), Z_n, which is the cumulative sum of the transformed variable z_i, is continuously plotted against the two stopping bounds *b* and *a*. It should be noted that the mean μ of the distribution is assumed to be known. Even when μ is unkown, the aforementioned procedure is still valid if x_i is replaced by y_i:

$$y_i = \left(\sum_{j=1}^{i} x_j - i\,x_{i+1}\right)\Bigg/\sqrt{i(i+1)} \quad \text{for } i = 1, 2, \ldots \tag{15}$$

It can be shown that now $\{y_i\}$ has i.i.d. normal distribution with zero mean and σ.

2.4. SPRT combined with extreme value statistics

Now, the SPRT is extended to the extreme values of the parent distribution, the distribution of the residual errors. In the previous section, the SPRT is formulated assuming that the residual errors have a normal distribution. However, slight errors in the normality assumption of the parent distribution can lead to larger errors for the extremes resulting in errneous false positive/negative indications of damage. To avoid this problem, the SPRT is reformualted using the probability distributions of extreme values. There are only three possible choices for the distributions of the extremes regardless the parent distrubution type: Gumbel, Weibull, or Frechet. Particulary, because the maxima of a normal distribution are known to have a Gumbel distribution and the resiual errors of the experimental study presented later are close to a normal distribution, the derivation presented here focuses on incorprating Gumbel distribution for maxima values into the SPRT. The cumulative distribution function for a Gumbel distribution is (Fisher and Tippet, 1928):

Gumbel $$F_M(x) = \exp\left[-\exp\left(-\frac{x-\lambda}{\delta}\right)\right] \quad \text{for} \quad \begin{array}{c} -\infty < x < \infty \\ \delta > 0 \end{array} \tag{16}$$

Similar formualtion of the SPRT can be easily derived for the other types of extreme value distribution and for minima values. For more information on extreme value statistics, please see *Extreme Value Statistics for Damage Detection in Mechanical Structures* by Worden et al, 2002.

Similar to Equation (11), the following hypothesis test is constructed using the standard deviation of the maxima as the parameter in question:

$$H_o : \sigma_M \le \sigma_o, \quad H_1 : \sigma_M \ge \sigma_1, \quad 0 < \sigma_o < \sigma_1 < \infty \tag{17}$$

Now, σ_M is the standard deviation of the residual error maxima, and the subscript "M" denotes a qunatity related to the maxima. σ_o is a user specified lower limit of the standard deviation for the undamage contion, and σ_1 is the other user specified upper limit for the damage condition. It is observed that the change of the maxima distribution's standard deviation is monotomically related to the change of the parent distribution's standard deviation. Here, an indirect statitstical inference on the standard deviation of the parent distribution (the distribution of the residaul errors) is conducted by examining the standard deviation of the maximum values.

It can be shown that the model parameters, λ and σ, of the Gumble distribution are related to its mean μ_M and standard deviation σ_M (Castillo, 1987):

$$\delta = \frac{\sqrt{6}}{\pi}\sigma_M \text{ and } \lambda = \mu_M - 0.57772\,\delta \tag{18}$$

If the distribution of the maxima is preprocessed such that the mean value is zero, Equation (12) can be rewritten in terms of λ and σ:

$$z_1 = \ln\frac{f(X_1 \,|\, \lambda_1, \delta_1)}{f(X_1 \,|\, \lambda_o, \delta_o)} \text{ and } z_i = \ln\frac{f(X_i \,|\, \lambda_1, \delta_1)\, f(X_{i-1} \,|\, \lambda_o, \delta_o)}{f(X_i \,|\, \lambda_o, \delta_o)\, f(X_{i-1} \,|\, \lambda_1, \delta_1)} \tag{19}$$

If $\{x_i\}$ are independent and identically distributed (i.i.d.), $f(X_i \,|\, \lambda_1, \delta_1)$ becomes $f(x_1 \,|\, \lambda_1, \delta_1) \times f(x_2 \,|\, \lambda_1, \delta_1) \times \cdots \times f(x_i \,|\, \lambda_1, \delta_1)$ and Equation (19) can be further simplified as follows:

$$z_i = \ln\frac{f(x_i \,|\, \lambda_1, \delta_1)}{f(x_i \,|\, \lambda_o, \delta_o)} \text{ for } i = 1,\ 2, \ldots,\ n \tag{20}$$

Next, the probability density function of the Gumbel distributionfor maxiam is obtained by differentiating the cumulative density function presented in Equation (16).

$$f(x) = \frac{dF(x)}{dx} = \frac{1}{\delta}\ \exp\left(-\frac{x-\lambda}{\delta}\right)\ \exp\left[-\exp\left(-\frac{x-\lambda}{\delta}\right)\right] \tag{21}$$

By substituting Eqaution (21) into Equation (20), z_i can be related to x_i:

$$z_i = -\ln\frac{\delta_1}{\delta_o} + \left(\frac{x_i - \lambda_o}{\delta_o}\right) - \left(\frac{x_i - \lambda_1}{\delta_1}\right) + \exp\left(-\frac{x_i - \lambda_o}{\delta_o}\right) - \exp\left(-\frac{x_i - \lambda_1}{\delta_1}\right) \tag{22}$$

By relating λ and σ to σ_M as shown in Equation (18), Equation (22) can be furhter simplied as follows:

$$z_i = -\ln\frac{\sigma_1}{\sigma_o} + \frac{\pi}{\sqrt{6}}\left(\sigma_o^{-1} - \sigma_1^{-1}\right)x_i + \exp\left(-\frac{x_i + 0.4504\,\sigma_o}{\sqrt{6}\ \sigma_o/\pi}\right) - \exp\left(-\frac{x_i + 0.4504\,\sigma_1}{\sqrt{6}\ \sigma_1/\pi}\right) \tag{23}$$

Finally, the cumulative sum of the transformed variable, Z_i, is monitored against the two stopping bounds, *a* and *b* calculated in Equation (8). A Z_i value less than *a* is indicative of acceptance of the H_0 hypothesis, while a Z_i greater than *b* indicates an acceptance of the H_1 hypothesis.

3. TEST STRUCTURE

The structure tested is a three-story frame structure model as shown in Figure 1. The structure is constructed of Unistrut columns and aluminium floor plates. The floors are 1.3-cm-thick (0.5 in) aluminium plates with two-bolt connections to brackets on the Unistrut. The base is a 3.8-cm-thick (1.5 in) aluminum plate. Support brackets for the columns are bolted to this plate and hold the Unistrut columns. The details of these joints are shown in Figures 2 and 3. Dimensions of the test structure are displayed in Figures 4 and 5. All bolted connections are tightened to a torque of 0.7 Nm (60 inch-pounds) in the undamaged state. Four Firestone air mount isolators, which allowed the structure to move freely in horizontal directions, are bolted to the bottom of the base plate. The isolators are inflated to 140-kPa-gauge (20 psig) and then adjusted to allow the structure to sit level with the shaker.

The shaker is coupled to the structure by a 15-cm-long (6 in), 9.5-mm-diameter (0.375-in) stinger connected to a tapped hole at the mid-height of the base plate. The shaker is attached at corner D, so that both translational and torsional motions can be excited.

Figure 1. Photo of the full test structure.

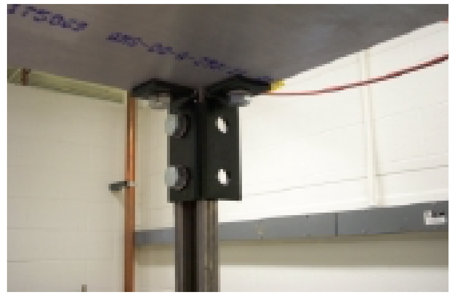

Figure 2. Photo of a joint on the structure.

Figure 3. Photo of the connection to the base plate.

3.1. Test Setup and Data Acquisition:

The structure is instrumented with 24 piezoelectric single-axis accelerometers, two spanning each joint as shown in Figure 5. Accelerometers are mounted on the aluminum blocks that are attached by hot glue to the plate and column. This configuration allows relative motion between the column and the floor to be detected. The nominal sensitivity of each accelerometer is 1 V/g. A 10-mV/lb-force transducer is also mounted between the stinger and the base plate. This force transducer is used to measure the input to the base of the structure. A commercial data acquisition system controlled from a laptop PC is used to digitize the accelerometer and force transducer analog signals.

Two damage cases are investigated in this experiment. The first damage is introduced to corner A of the first floor (Damage 1) and the second damage is placed at corner C of the third floor (Damage 2). These two damage locations are shown in Figure 1. For each damage case, the bolts were loosened until hand tight, allowing the plate and column to move slightly relative to one another. After the damage cases, all the bolts were tightened again to the initial torque of 0.7 Nm (60 in-pounds). Five time series are measured from the initial undamaged case, and these time series are used for constructing the reference database, or training. Five time series are recorded under each damage case, and additional five time series are obtained after tightening all bolts to the initial torque values. These time series are then used for testing the proposed SPRT procedure. A total of 20 time series are used for this experiment.

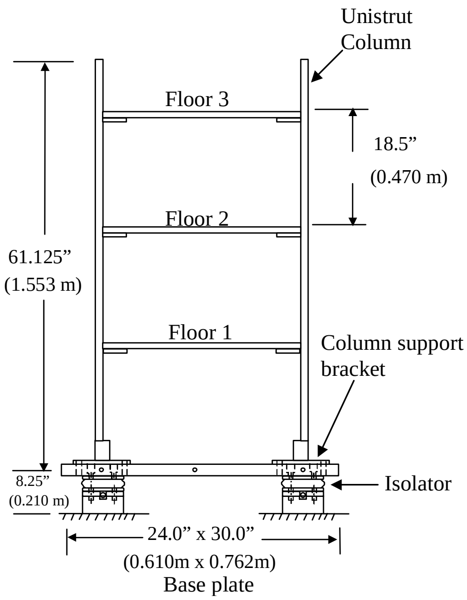

Figure 4. Basic dimensions of the three-story frame structure.

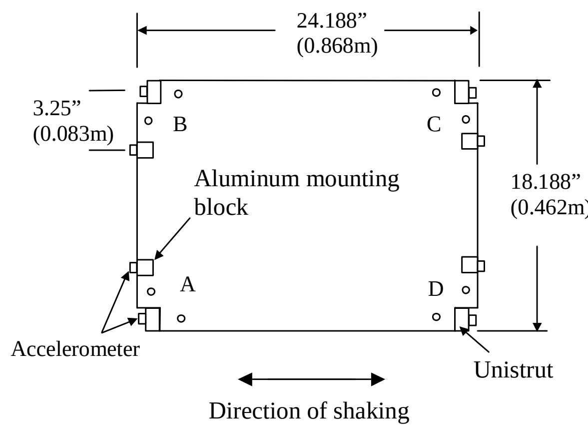

Figure 5. Floor layout as viewed from above.

4.RESULTS

The time series analysis begins with the assumption that a “pool” of signals is acquired from a known structural condition of the system. In this example, multiple time series are recorded from the undamaged structure. The collection of these time series is called the “reference database” in this study. The construction of this reference database is shown to be useful for normalizing data with respect to varying operational and environmental conditions. The applications of this time series analysis to data normalization are presented in Sohn and Farrar (2001) and Sohn et al. (2001).

Instead of independently analyzing 24 time histories from each accelerometer, the point-by-point difference between time series from the two adjacent accemeroters at a joint is first computed. Then, the resulting 12 time series corresponding to each joint are used for the AR-ARX modeling. The order r in the AR model is set to 25, and the p and q orders for the ARX model are set to 20 and 5, respectively. Satisfactory prediction errors mostly less than 10% error are achieved for all the reference signals indicating that the selected AR-ARX model aprropriately characterizes the underlying dynamic system of each signal readings.

A linear prediction model combining AR and ARX models is employed to compute the damage-sensitive feature, which in this case is the residual error between the prediction model and measured time series.

Next, the SPRT-N (normal distribution formulation) and SPRT-G (Gumbel distribution formulation) are applied to the residual errors obtained from the AR-ARX modeling. The type I & II errors are set to 0.001. The formulation of the sequential probability test here is based on the premise that, when a system being monitored undergoes a structural change such as damage, a signal measured under the new structural condition will be significantly different from the signal obtained from the initial undamage case. Therefore, when a time prediction model is constructed using the baseline undamaged time signal, the prediciton error of the newly obtained signal, which is again from the dacasmage case, will departure from that of the baseline signal. Particularly, the prediction error of the new singal is expected to increase. Based on this observation, the sequential hypothesis test is cast as follows for SPRT-N:

$$H_o : \sigma \le \sigma_o, \quad H_1 : \sigma \ge \sigma_1, \quad 0 < \sigma_o < \sigma_1 < \infty \tag{24}$$

In this particular example, σ_0 and σ_1 are set to 0.40 and 0.42, respectively. Note that the establishment of the σ_0 and σ_1 values is based on the observation of actually damage cases. That is, changes of the standard deviation should be first monitored for the corresponding damage cases to select the appropriate σ_0 and σ_1 values. This selection of the σ_0 and σ_1 values categorizes the proposed method as a supervised learning method. In a similar fashion, the sequential hypothesis test for SPRT-G is cast as follows:

$$H_o : \sigma_M \le \sigma_o, \quad H_1 : \sigma_M \ge \sigma_1, \quad 0 < \sigma_o < \sigma_1 < \infty \tag{25}$$

σ_0 and σ_1 are set to 0.24 and 0.26 based on the similar obervations as before.

The results of damage classification using SPRT-N and SPRT-G are reported in Table 1 and 2. To briefly summarize the results, both methods illustrate comparable performance. SPRT-N and SPRT-G do not show any false-positive indications of damage for all five undamaged cases. For the first damage case (Damage 1), the damaged joint is located at the corner A on the first floor, and this joint is associated with sensor readings from channels 17 and 18. Using SPRT-N and SPRT-G, the correct damage location is correctly revealed for all five cases of Damage 1. For the second damage case (Damage 2), where the bolts at the corner C on the third floor is hand tight and this joint corresponds to channel 5 and 6 readings, SPRT-N indicates that the adjacent joint at the corner D on the same floor is most likely damaged. SPRT-G also suggests the existance of damage at the same adjacent joint but correclty identifies the actually damaged joint 3 times out of the five exampled time series.

Table 1: Damage classification results using SPRT-N (normal)

Test Case	Ch1-Ch2	Ch3-Ch4	Ch5-Ch6	Ch7-Ch8	Ch9-Ch10	Ch11-Ch12	Ch13-Ch14	Ch15-Ch6	Ch17-Ch18	Ch19-Ch20	Ch21-Ch22	Ch23-Ch24
Undamaged	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
Damage 1	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0
Damage 2	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0

*The zero '0' denotes that the null hypothesis is accepted indicationg no damage is present at that joint, and the unity '1' denotes that the null hypothesis is rejected and the corresponding joint is damaged. The shaded areas represent the locations of the acutally damaged joints, and the hypothesis results in these shadded ares should ideally correspond to 1. The hypothesis results should be zero otherwise.

** For each undamaged and damage cases, five time series are recorded, and the corresponding damage classification results are shown.

Table 2: Damage classification results using SPRT-G (Gumbel)

Test Case	Ch1-Ch2	Ch3-Ch4	Ch5-Ch6	Ch7-Ch8	Ch9-Ch10	Ch11-Ch12	Ch13-Ch14	Ch15-Ch6	Ch17-Ch18	Ch19-Ch20	Ch21-Ch22	Ch23-Ch24
Undamaged	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
Damage 1	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0
Damage 2	0	0	1	1	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	1	1	0	0	0	0	0	0	0	0
	0	0	1	1	0	0	0	0	0	0	0	0

*The zero '0' denotes that the null hypothesis is accepted indicationg no damage is present at that joint, and the unity '1' denotes that the null hypothesis is rejected and the corresponding joint is damaged. The shaded areas represent the locations of the acutally damaged joints, and the hypothesis results in these shadded ares should ideally correspond to 1. The hypothesis results should be zero otherwise.

** For each undamaged and damage cases, five time series are recorded, and the corresponding damage classification results are shown.

5.CONCLUSIONS

In an effort to find an automated and quantitative method for damage identification, a unique integration of time series analysis, statistical inference, and extreme value theory is explored. Time series analysis techniques, solely based on the measured vibration signals, are first employed to extract damage sensitive features from a structure for damage classification. While there has been increasing interest in the filed of structural health monitoring, the decision as to whether a structure is damaged or not tends to be made on the basis of exceeding some heuristic threshold. In this study, the sequential probability ratio test (SPRT) is employed to provide a more principled statistical tool for this decision-making procedure, excluding unnecessary interpretation of the measured data by users. Finally, the performance and robustness of damage classification is improved by incorporating extreme values statistics of the extracted features into the SPRT. The applicability of the SPRT to structural health monitoring is demonstrated here using measured time signals from a three-story frame structure tested in a laboratory environment. The framework of the proposed SPRT method is well suited for developing a continuous monitoring system, and can be easily implemented on digital signal processing (DSP) chips automating the damage classification process.

ACKNOWLEDGMENTS

This research is funded through the Laboratory Directed Research and Development program at Los Alamos National Laboratories.

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